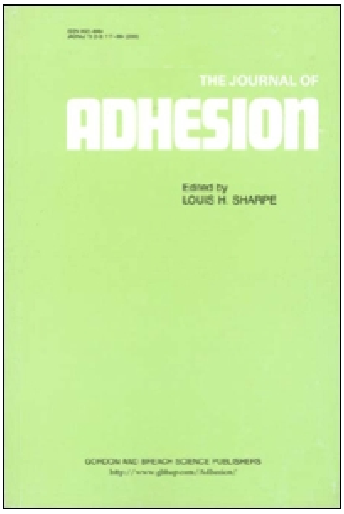


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# NOTE

# The Optimum Profile for a Lap Joint.

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IN LAP JOINTS, stress concentrations in the adhesive layer can arise from differences in elastic moduli and abrupt variations in thickness of the adherends and the adhesive layer[1]. Various attempts have been made to design joints in which these stress concentrations are minimised. Mylonas and de Bruyne[2] suggested that the stress concentrations could be reduced by tapering the ends of the adherends, so that a more even distribution of strain along the joint could be obtained. Hennig[3] suggested that the same objective would be attained by using a high modulus adhesive in the centre of the joint and a lower modulus adhesive at the ends of the joint; he reported that an increase in joint strength of 20% could be obtained by this method. Segerlind[4] discussed the variation of the magnitude of the stress concentrations in a lap joint with the dimensions of the joint and showed that since the increase in joint strength with increase in overlap effectively fell to zero above a given length of the overlap (dependent upon the geometry of the joint and the physical properties of adhesive and adherend), then it was possible to specify an optimum overlap for the joint.

The object of the present note is to show that by correct choice of the profiles of the adherends it is possible to manufacture a lap joint for which the shear stress in the adhesive layer is uniform throughout the joint; that is, there are no shear stress concentrations in the adhesive and the maximum load carrying capacity of the adhesive is developed.

Fig. 1. shows two components which are to form the adherends of the lap joint. These are of unit width perpendicular to the plane of the paper and of arbitrary profile. The planar faces AB and DE are of length a.

If these components are subjected to a system of forces let $e_{1x}$ be the tensile strain parallel to AB at a point in the face AB distant x from A when adherend 1 is unstrained, and let $e_{2x}$ be the corresponding strain in the face DE.

If in a lap joint made from these adherends, the adhesive layer allows negligible displacements across its thickness, then $e_{1X}$ must equal $e_{2X}$ at opposite points on the two interfaces. If the adhesive layer is of uniform thickness and shear modulus, but sufficiently thin for edge effects[5] to be neglected, and if the shear stress in the adhesive is to be constant, all parallel lines in the unstrained state must remain parallel in the strained state, and again $e_{1X}$ must equal $e_{2X}$ at all points along the adhesive layer. This information can be used to find suitable profiles for the adherends. In Fig. 1. uniform shear stresses $\tau$ act on the surfaces AB, DE, of the separate adherends, and are balanced by forces **F** acting across the faces CB and DF. The combination of the force **F** and the shear stress $\tau$ acting on each adherend results in a moment which can be counteracted by an equal and opposite moment acting at the faces; we shall suppose that these moments are provided by distributed forces acting perpendicular to the faces AB and DE. With this loading, the tensile strains $e_{1X}$ and $e_{2X}$ depend on the profiles AC and FE; by adjustment of these profiles $e_{1X}$ and $e_{2X}$ can be made equal at corresponding values of x. If adherends in which $e_{1X}$ and $e_{2X}$ are equal are formed into a lap joint, then on application of the forces **F** the shear stress at the interfaces will be uniform. There will however, be tensile stresses at the interfaces which are not uniform.

In order to find these profiles, consider the equilibrium of elements AGH and EJK when there is a uniform shear stress $\tau$ applied to the faces AH and KE.

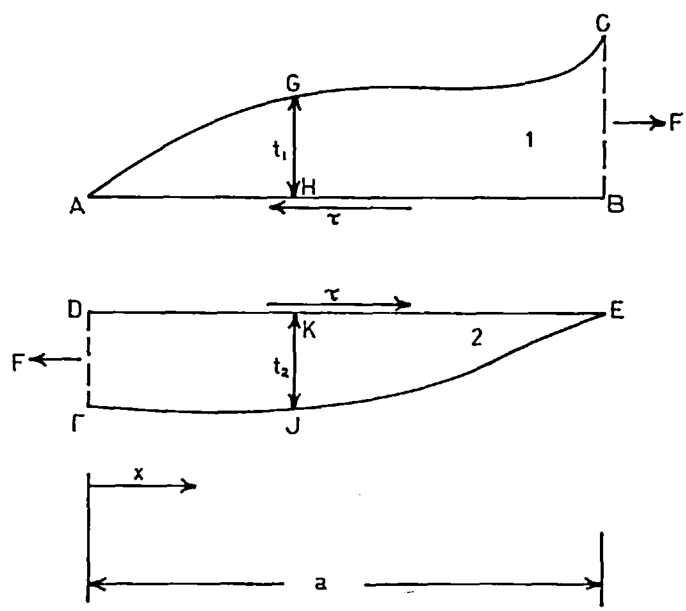


**Figure 1.** Representation of the distribution of uniform shear stresses acting on the surfaces of the separate adherends.

If $t_{1X}$ and $t_{2X}$ are small compared with a,

$$E_1 t_{1X} e_{1X} = \tau x \qquad (1)$$

and

$$E_2 t_{2X} e_{2X} = \tau (a-x) \qquad (2)$$

where $E_1$, $E_2$ are the elastic moduli, and $t_{1X}$ and $t_{2X}$ are the thicknesses at distance x from the origin in the unstrained state in adherends 1 and 2, respectively. The shear stress across the adhesive/adherend interfaces will be uniform if $e_{1X} = e_{2X}$, i.e., if:

$$E_2 t_{2X} x = E_1 t_{1X} (a-x) \qquad (3)$$

Hence equation (3) represents the design condition for a uniform shear stress across the adhesive/adherend interfaces. Furthermore, if the thicknesses of the adherends are small compared with the length of the overlap, the tensile stresses acting perpendicular to the interfaces are negligible, so that profiles designed in accordance with equation (3) will result in the development of the maximum strength of the joint.



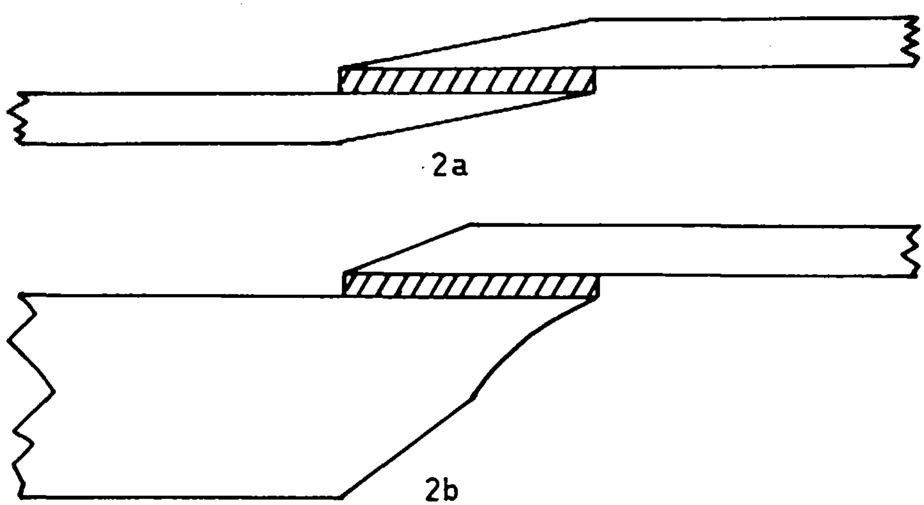


**Figure 2.** Profiles of bonded joints calculated from Equation (3) to optimise joint strength.

In any component incorporating a lap joint, the shape and modulus of at least one of the adherends is usually determined by other requirements of the system. Figures 2a and 2b show profiles calculated according to equation (3) to optimise the joint strength in two cases. In Fig. 2a, the moduli of the two adherends are taken as equal, and one adherend has a uniform taper extending over the length of the overlap. The configuration shown in Fig. 2a is the same as that proposed on intuitive grounds by Mylonas and de Bruyne[2].

In Fig. 2b, one of the adherends is tapered over only part of the region of overlap, and its Young's modulus is twice that of the other. It can be seen that the maximum thickness of the low modulus adherend is much greater than that of the other. Differentiating Eqn. (3) with respect to x and putting x and $t_{10}$ equal to zero, we find that

$$t_{2x} = \frac{E_1}{E_2} a \frac{dt_{1x}}{dx} \tag{4}$$

That is, the greater thickness results from both the lower modulus of this adherend and the increased angle of the taper at the end of the other.

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